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Analyses Of Scale Free Models And Inter-Disciplinary Applications

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Abstract: Scale Free Networks have emerged as a promising field of network science. These networks are topologically evolving networks where stochastic processes govern the addition of new nodes and links and removal of old ones from the network. Recently, the scale free topological features have been observed in diverse and broad range of networks across the world which has surged interest in this field of network science. Though scale free networks are so pervasive, yet very little is known about their structure and behaviour. This paper discusses the basic characteristics of scale free networks and critically analyses various scale free architectural models and their applications across different fields. Thereafter, few open research areas are discussed to offer opportunities for future research directions.

Keywords: Scale Free Models, Preferential Attachment, Power Law, Barabasi-Albert Model.

1. Introduction

For decades, all complex networks such as Internet, World Wide Web, Citation networks and Cellular networks were regarded as random networks as suggested by Erdos and Renyi [1][3]. It was only in the late 1990s, when Barabasi and Albert mapped degree distribution of nodes' in real networks such as World Wide Web [2], the concept of scale free networks emerged. A significant deviation from random behaviour was observed, it was found that there are a few nodes which have very huge degree while others' have very few connections. The high-degree nodes are called the hub nodes in the network. The degree distribution of nodes' in scale-free networks and random networks is shown in Figure 1. Hubs are not allowed in random networks while there are a few large hubs in scale-free networks. Nodes in scale free network follow power-law distribution while random network nodes' follow Poisson distribution.

The power-law node distribution is a slow decreasing function with a heavy tail. In such networks, few hubs dominate and play a significant role in deciding network properties. Thus, equality among nodes does not exist in scale-free networks. This over-reliance on hubs has its own merits and de-merits which will be discussed in the subsequent sections of this paper.

The first and most basic model of Scale-free networks is Barabasi-Albert Model proposed in 1999. According to this model, there are two factors which govern the development of scale-free networks, namely, growth and preferential attachment [7]. The network keeps on growing with the addition of new nodes which preferentially attach themselves to the node which has the highest degree. The literature is replete with several other variants of this model which are discussed later in the paper. Scale-free models exhibit self-similarity which implies that any subset of the network is almost similar to the rest of the network.

In the past few years, network architecture similar to scale-free networks has been observed in diverse and complex fields such as cellular networks, geophysical networks, metabolic networks, technological networks, communication networks, social networks and citation networks [4- 13]. All these networks possess similar statistical characteristics and obey common evolutionary laws. This paper systematically studies the ubiquitous presence of these features to alter the way we approach and study the complex networks around us.

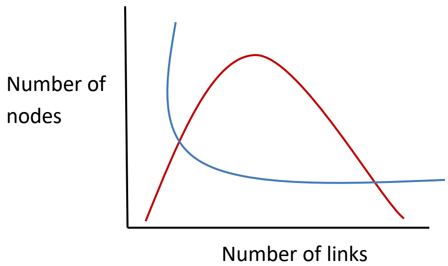

Figure 1: Degree distribution in random and scale free networks

The rest of the paper is organized as follows. Section 2 discusses some of the important features of scale free networks. In section 3, the potential implications of scale free networks across different disciplines are presented. Section 4 discusses some of the important scale free models proposed so far. Conclusions and future research directions are given in section 5.

2. Characteristics of Scale Free Networks

Scale free network is a self-organised evolutionary network where the interaction of individual nodes determines the collective properties of the network.

The important characteristics of these networks are discussed below:

- **Adherence to power-law:** The degree distribution of nodes in scale free networks follow power-law, $P(k) \sim k^{-\gamma}$, where K is node degree and γ is degree exponent. The value of γ varies between 2 and 3.
- **Small Average path length:** These networks have small average path which grows logarithmically with network size. This feature reduces network delay and achieves high spectral efficiency which increases network lifetime.
- **High Clustering Coefficient:** These networks have high clustering coefficient. The clustering coefficient of a node is defined as the ratio of the number of actual connections between its neighbours to the maximum possible connections between

them as shown in Figure 2. The formula for finding the clustering coefficient of a node i is, $C_i = \frac{2E_i}{K_i(K_i-1)}$ where E_i is the number of actual links among neighbours of node i and K_i is the total number of neighbours of node i.

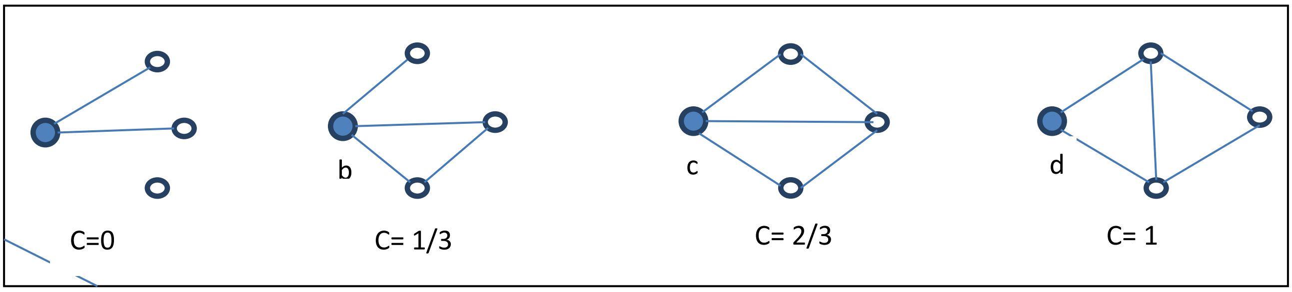

Figure 2: Example of clustering coefficient. Node a, node b, node c and node d has clustering coefficients of 0, 1/3, 2/3 and 1 respectively

- **Follow Scale Free property:** The nodes in the network obey scale free property which states that the ratio of hub nodes to the other nodes in the network remain same irrespective of network size.
- **Short Diameter:** The diameter of scale free networks is short and grows slowly with network size.
- **Scale-Invariant:** These networks are scale-invariant. The dynamic additions of nodes/ links in the network do not affect the performance of scale free network.
- **Robust:** The likelihood that a random removal of nodes will affect performance of scale –free network is very small. Although these networks are very robust against random attacks, these are very prone to coordinated attacks on hubs of the network.

3. Scale-Free Networks across disciplines

In today's world, networks have become so pervasive and strongly coupled that they cannot be studied in isolation. The inter-dependence of Business Networks, Trade networks, Social networks and the World Wide Web is an example. Almost all of these networks share characteristics such as modular topology, heterogeneous nodes, disassortative connections, and similar large-scale topology power-law law nodes' distribution. The existence of scale-free property among these networks with different purpose and function is quite intriguing and need to be studied extensively. Some of the real-world examples where the scale-free property has been discovered in the last two decades and its implications are discussed below and the same has also been presented in table [1].

3.1 Economics: The famous 'Rich Gets Richer' and "First Movers Advantage' phenomenon in economics is somewhat similar to preferential attachment in scale-free networks. Recently, scale-free networks have emerged as a substantive tool to explain different economic networks such as Business networks [11], supply chain networks [23][28], International trade networks [9], Banking networks [10] and Marketing [5]. The nodes are the various economic entities such as banks, markets, products, ideas, innovations and policies. The links represent different interactions such as firm partnerships, financial transactions and bank/companies' mergers.

Today, there is economic synchronization among all the countries of the world which creates ripple effects by propagating economic disruptions throughout the world economy such as 2001 Indian stock market scam, 2008 financial crisis, 2010 flash crash, 2015 Chinese market crash. By analysing the network characteristics and evolution, important hubs can be identified. This visualization can help in measuring the risks and taking preventive measures to avoid such disturbances. The scale free property also helps in understanding the impact/diffusion of ideas and policies on the national economy.

World Trade Web: The international trade relations can be described as world trade network where nodes represent countries and links represent their trading partners. M. Angeles Serrano et al [9] analysed the topological features of world trade web and found scale free degree distribution and high clustering coefficient between trading countries. The small countries prefer to have trading pacts with central trading countries such as China, USA, and Japan due to presence of competitive prices and desirable products.

The high reliance on trade hubs has some negative implications also. The 2008 Financial crisis which originated in USA (trade hub) due to failure of investment Bank Lehman Brothers propagated its effects throughout global economy and delayed economic growth of several countries.

Supply Chain Networks: Global supply chain networks can be regarded as scale- free where firms, companies, producer, distributors and retailers can be represented by nodes and links represents the interactions in the form of financial transactions or exchange of material between them. The fitness-based scale-free model is suitable for supply chain networks [23]. Whenever a new agent enters the networks, some fitness parameters such as cost, service quality, firm size, market perception and geographical location are considered before joining any node. The scale free features have been in observed in the Indian auto component industry [27], Airbus SCN [25], Agricultural products [26] and Chinese automobile industry [28].

3.2 Social Network: The relationships among individuals in society can be represented as a graph where nodes represent people and connections or social relations between them represent links. This network keeps evolving as nodes and links are being added or removed. One such social network example where scale free properties have been identified recently is Facebook Page Network (FPN) and its BPO-BPO sub-network [31] where the number of likes a page receive is denoted as its node degree. A sample of 4.8 million nodes was studied and power-law features were found with λ= 1.53 for FPN and λ=2.25 for BPO network. Another popular example is Twitter [32], where followers link distribution follows power-law with exponent λ= 2.17 for outdegree and λ= 1.87 for indegree distribution.

3.3 Epidemic Network: A proper understanding of the scale free networks can provide new insights on controlling spread of computer viruses and spread of contagious diseases such as HIV, Ebola virus, Dengue, SARS and Polio. Liljeros et al [8] discovered scale free features among the sexual contacts of humans where sexual hubs play a key role in propagating viruses to a large proportion of population. The degree distribution of dengue epidemic in Singapore [30] and Malaysia [29] also exhibited features of scale free networks. Recently, various COVID 19 infection spread models [33][34] have been developed with the concepts of scale free networks. This discovery is very beneficial for government agencies in identifying super spreader sites and take policy decisions accordingly. The basic epidemiological model states that if the transmission probability of a disease is under epidemic threshold value then the disease can be contained. Instead of random immunization, we must identify and immunize hubs whose degree is larger than a given degree k_0, so as to achieve epidemic threshold. A field where epidemics are encouraged is marketing business where viral campaigns are organized to promote new products, ideas, medicines and policies of the government. The knowledge of how information is being propagated on scale free networks can provide innovative ways to market products so as to reach more customers.

3.4 Citation Network: In 1965, Price studied the paper citation pattern and gave the concept of "success breeds success" or cumulative advantage distribution which is quite similar to the preferential attachment mechanism of scale-free networks. The citation network was later explored efficiently through network science concepts and was found to obey scale free property where the published papers represent node and link represent citations. One of the implications of such findings is that well-cited papers are more expected to be cited frequently. The parameters which contribute to the attractiveness of a paper are popularity of author, journal indexing and relevance of ideas and importance of work in today's context. Wang Ming-Yang et al [20] proposed a short-term preferential attachment mechanism for citation network where preferential probability is dependent on the number of times paper has been cited in last one year.

3.5 Internet and WWW: Scale-free topological features have been identified in both World Wide Web [2] and Internet [3][7][35] where nodes represent WebPages and routers respectively. The identification of power-law distribution and preferential attachment mechanisms has been extremely beneficial in understanding the dynamics of Internet and to model its topological structure. The power-law distribution in the topological structure of internet at router and autonomous system level was initially identified in [35] and further studied in detail in [36]. It has also been observed that the relation between sub-networks and router interface captured using bipartite network topologies also features scale-free characteristics.

3.6 Computer Science: Scale-free networks can help in designing realistic models for simulations, traffic engineering and designing network protocols. Linfeng Liu et al [16] used the concepts of scale-free model to develop a topology construction model for acoustic sensor networks. Various scale-free models for wireless networks have been proposed so far such as Neighbourhood Log-on Log-off model[42], Flow Aware Scale

Free model[45], Energy-Aware Barabasi model [38] and Local Area and Energy Efficient model (LAFE) [43].

The concepts of scale-free networks have also been applied in software engineering. Lian Wen et al [22] studied the component dependence network of several libraries and applications in java. The author also explored sorting comparison network of different sorting algorithms where node represents the record and link denotes the inter-record comparisons. The scale free features were found in both the networks. This discovery is extremely beneficial in identifying important components during software maintenance, web mining and enhancing software security.

Table 1: Examples of Scale-Free Network

FIELD	NETWORK	NODES	LINKS	IMPLICATIONS
Economics	International Trade Network	Countries	Trading partners	Economic dependency
	Business Network	Companies/Industries	Partnership with firms	Promotion of new products, ideas and policies
	Supply chain networks (SCNs)	Entities such as producer, distributors, retailers	Interactions among entities	Robustness enhancement
	Banking Network	Banks	Financial transactions	Control cascading failures, enhance reliability and security
Scientific collaborations	Citation Networks	Scientific paper	Citations	Improved readability
Epidemics	Communicable disease Network	People	Sexual relationship, Social relationship	Spreading of biological viruses, effective policy decisions, targeted immunization strategy
	Digital Network	Products/People	Marketing campaigns/Social interactions	Improved information dissemination

Sociology	Social Networks	People	Interactions among people	Rumour and Memes Spreading, Knowledge diffusion
Computer Science	Software Engineering	Records, libraries, applications	Dependence relationship	Reverse engineering, web mining, software maintenance
	Sensor Networks	Sensor nodes	Sensor nodes connections	Effective routing decisions, improved network reliability
	Internet	Routers	Wired/Wireless connections	Routing efficiency, Malware spreading, Avoid cascade failures
	World Wide Web	WebPages	Hyperlinks	Efficient Traffic distribution, Improved network lifetime

4. Scale Free Models

A meaningful network model provides a good understanding of real world networks and helps in retrieving useful information from it. The way a network evolves with time plays an important role in developing understanding of the underlying network structure. Models help in measuring the performance of critical network functions such as topology generation, routing, congestion control and security. Some of the scale free models for constructing scale free networks are discussed below:

BA Model: The first and simplest model of scale free networks is given by Barabasi and Albert [1] in 1999 called BA model or the scale-free model. Initially, network consists of m_0 nodes arbitrarily linked to each other. The network evolution process is governed by two mechanisms:

a) Growth: At every timestamp, a new node with m links where $m \leq m_0$ enters the system and connects to the existing nodes in the network.

b) Preferential attachment: The probability $\prod(k_i)$, that a new node connects itself to an existing node i in the network depends on degree of node (K_i) according to equation (1).

$$\prod(k_i) = \frac{k_i}{\sum_j k_j} \tag{1}$$

This implies that new nodes in the network prefer to connect to nodes with higher degree. Both growth and preferential attachment are quintessential for scale free property. This model further reinforces the Rich get richer and First movers' advantage phenomenon observed largely in society. The degree exponent γ value predicted by BA model is 3. The preferential attachment function is a slow decreasing probabilistic function. Figure 1 shows scale free network evolution with m=2.

The increase in the degree of nodes' in BA model is sub-linear as competition among existing links increase with the addition of each new node in the system. The scale free network evolution in a circular layout is shown in Figure 3.

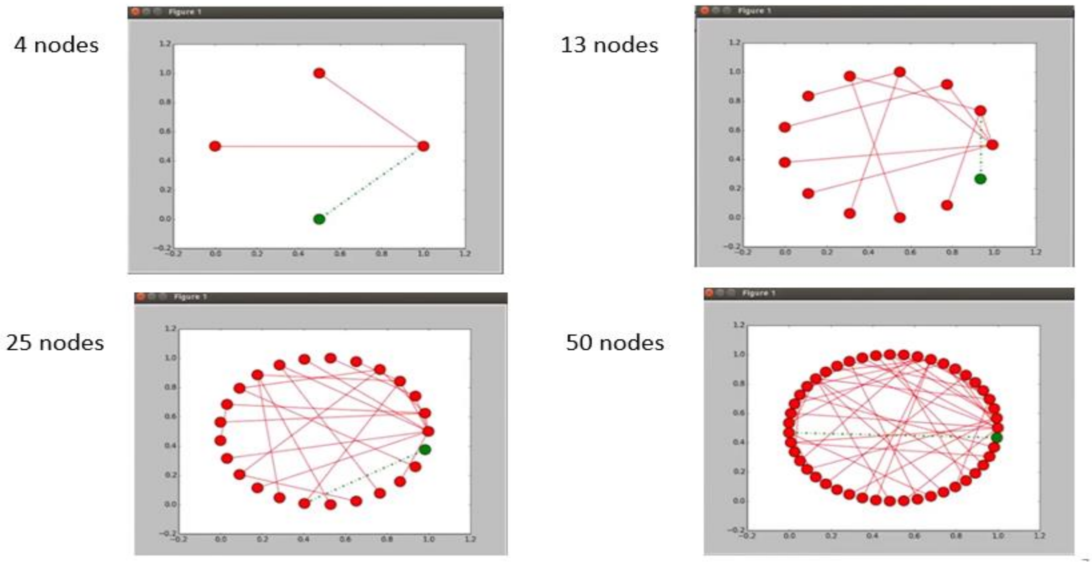

Figure 3: Scale Free Network evolution

Being a minimal model, there are many limitations of BA model which are listed as follows:

- It predicts degree exponent to be 3 while experimental value varies between 2 and 5.
- The first mover advantage concept given by BA model is not always applicable as age is not the only factor that affects the growth rate of a node. For example: Google was launched in 1998 much later than Alta Vista launched in 1995, still it is the leading search engine. This implies there are some other factors such as creativity, genetic reasons; quality of service offered etc. which affects the growth of a network. The pure power-law degree distribution is rarely observed in nature.
- Nodes/links in a network can die as the network expands. They can also be deliberately removed or get accidently damaged with time. For example, in wireless sensor networks many sensor nodes die after exhausting their energy or service record of an employee in an organization is deleted if he/she leaves or is fired from organization. The dynamic addition and removal of nodes is an implicit part of any network structure.

To account for deficiencies of BA model, many other models have been proposed. Some of them are discussed below:

Fitness driven attachment model: Bianconi-Barabasi gave the concept of fitness driven attachment model [6] also called fitness model. Fitness corresponds to the collection of all attributes of a node that determines its ability to attract new nodes in the network. Each node i have a fitness value denoted by η_i which remains constant. The BA preferential rule is modified to include fitness parameter as shown in equation (2). This means a node with high fitness will attract new nodes faster than other nodes and may even surpass nodes which arrived earlier in the network. A real life example of this model is Facebook which became far more successful than its predecessors such as orkut.

$$\prod i = \frac{\eta_i k_i}{\sum_j \eta_j k_j} \quad (2)$$

Lognormal Fitness Attachment Model: It is implausible that when a new node enters the network it has information about node degree of all the existing nodes in the network. For example in Sexual Network, the number of sexual contacts of an individual is not known to everyone and is thus not suitable for preferential attachment. On the other hand, the sexual preference of an individual is not dependent only on the number of sexual contacts another person had. Ghadge et al [21] postulated a generative network model only based on fitness of a node where multiple factors contribute to fitness. The preferential attachment rule for a new node i with m edges as proposed by Ghadge is given in equation (3)

$$\pi_i = \frac{\rho_i}{\sum_{j=1}^{n'} \rho_j} \quad (3)$$

Where, ρ_i is the fitness of node i and n' is the number of nodes currently in the network.

Initial attractiveness model: According to BA model, the probability of acquiring links for a remote/isolated node is exactly zero. But in real life, there are chances that even a new actor gets role in some movie or a paper gets cited for the first time. To incorporate this fact, initial attractiveness model modifies preferential attachment function as given in equation (4)

$$\prod(k) \sim A + K \text{ , where A is a constant.}$$
(4)

Double preferential attachment model: The links in the network are established due to not only new nodes but also existing nodes in the network. The new links between existing nodes in the network are called internal links. For example, www and social networks. In such cases double preferential attachment function is used to represent probability of establishment of internal links between nodes with degree k and k' written as equation (5)

$$\prod(k.k') \sim (A + BK)(A + Bk') \quad (5)$$

Random attachment model: Xu-Hua Yang et al [40] showed how random attachment can generate a scale free network in nature. The proposed model does not follow preferential attachment rule and requires no topology information either local or global. Every time a new node enters the network, it selects one of the random neighbours and further elects one random vertex of that selected neighbour to attach as shown in Figure 4.

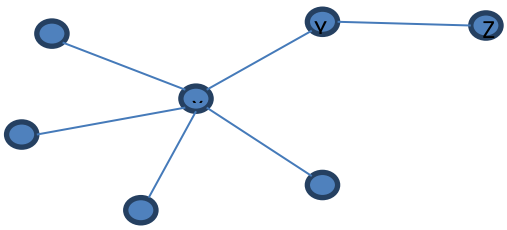

Figure 4: Random attachment model

Link selection model: Another local and random model is link selection model where nodes do not follow preferential attachment to build scale free networks. When a new node enters the network, it selects a link randomly and connects itself to any of its two ends as shown in Figure 5. The preferential attachment function is absent in this model.

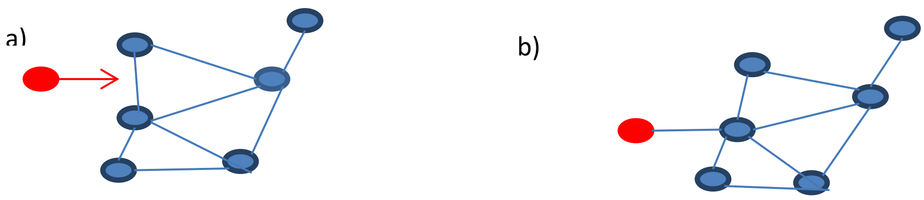

Figure 5: Link selection model

Copying model: In this model, a new node chooses a target node randomly. After selecting the target node, node has two options. With probability p, It can connect itself to the target node or with probability 1-p, It can choose an outward link of target node and connect to that link's end node as shown in Figure6. The model is quite appropriate in real world networks. Scientist or research scholars while studying a specific topic often copy references from the papers, they have read to get in-depth knowledge of that topic.

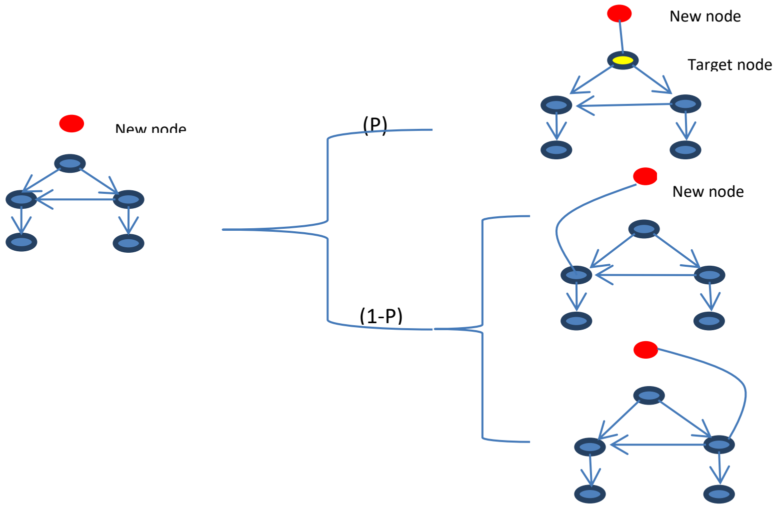

Figure 6: Copying model

Local preferential attachment model: Li-Na Wang et al [15] proposed a scale free model based on local preferential attachment. The idea is that nodes in a network usually prefer to connect to other nodes in their local area due to either limited information or close relations. A node along with all its neighbours forms a local area network represented as L. The new node x perform local preferential attachment and attaches itself to a node y from neighbourhood L according to equation (6). α is an adjustable parameter.

$$\prod(k_x) = \frac{k_x^{\alpha}}{\sum_{y \epsilon L} k_y^{\alpha}} \tag{6}$$

Group preferential model: Bing Ye et al [18] gave group preferential model where at each time step a new node i attaches itself to a group of m vertices represented by $A_{(x_{i,}x_{2,}\ldots\ldots x_{m})}$ in the network selected according to preferential probability $P(A_{(x_{i,}x_{2,}\ldots\ldots x_{m})}(t))$ given in equation (7),

$$P(A_{(x_{i,}x_{2,}\ldots\ldots x_{m})}(t)) = \frac{K_{i_1}(t) + K_{i_2}(t) + \cdots \ldots\ldots k_{i_m}(t)}{C_{n_t-1}^{m-1}\, N_t} \tag{7}$$

Scale free model for wireless networks: The robustness and connectivity offered by scale free networks have attracted the researchers working in the field of wireless networks. Recently, many attempts have been made to apply scale free property in generation and operation of wireless networks [39]. The nodes in wireless networks have limited transmission power which restricts the area over which they can send packets, thus the assumption of global preference does not hold true in these networks. The node degree of all the nodes in the region is not known to new node. Young-bin Kim et al [37] proposed a scale free model for wireless networks where a feedback threshold value is calculated. Whenever a new node enters the network, only those nodes whose degree is more than the feedback threshold advertise their degree information. The feedback threshold varies with time to capture real time degree information with increasing network size.

Neighbourhood log-on and log-off model: Due to limited energy, frequent topology changes and harsh deployment conditions, nodes in wireless sensor networks die out frequently. Thus it is impertinent for scale free wireless sensor network model to include node/link deletion in topology evolvement. Yue Wang et al [42] postulated a Neighbourhood log-on and log-off model that considers both node addition and removal during network topology generation. Usually, new nodes prefer to link to nodes in their local neighbourhood. A set of randomly chosen vertices form neighbourhood of a node. New node with m edges connects itself to priority vertices selected from the neighbourhood. A compensation mechanism is added to quantify connectivity of the network. When a node i lose an edge, it initiates s new edges. Upon deletion of chosen node, if any other node loses edges then node i will get preferential attachment.

Flow Aware Scale Free Model (FASF): Dong Wang et al [45] propounded a Flow Aware Scale Free (FASF) for WSNs which considers the energy consumed by nodes in the both network generation and network operation. Each edge of the network has a weight value associated with it. Weight of a node is the total sum of the weight of all its edges. The probability of selection of an existing vertex p depends on its degree k_p in such a way as given by equation (8)

$$\prod(k_p) = \frac{k_p}{\sum_q k_q} \tag{8}$$

After selection of node p with degree k_p, the network is further explored to find out all the nodes with degree k_p and then one with the minimum weight is selected as the final vertex. When new node p attaches itself to an existing node q, the traffic created by node p has more influence on the immediate neighbourhood of node q rather than the whole network, thus weight is incremented accordingly. The model also compensates for the frequent node/link failure in WSNs by adding few new edges for every link removed in the network. The model balances the energy consumption in the network, thereby increasing network lifetime of WSNs.

Scale free model for wireless sensor networks: Hongxing Yao et al [41] postulated another scale free model for wireless sensor networks which takes into account node/link addition, node/link removal and the residual energy of nodes while generating topology of the network. The preferential attachment equation of the model is given in (9) where E_i and K_i represent energy and degree of node I,

$$\prod\nolimits_1(K_i) = \frac{E_i K_i}{\sum_j^N E_j K_j} \tag{9}$$

Energy Aware Barabasi-Albert (EABA) model [38]: This model gives different weights to node degree and residual energy of nodes during topology evolution by introducing tunable coefficients a and b as given in (10)

$$\prod(k_i) = \frac{a.k_i}{\sum_n k_n} + \frac{b.E_i}{\sum_n E_n} \tag{10}$$

Where a+b=1, both a and b are greater than or equal to zero, K_i and E_i are the degree and residual energy of node i respectively. By varying the value of coefficients a and b, preferential attachment equation can be adjusted according to the constraints and resources of networks across a broad range of fields.

Zhang Xuyuan et al [14] gave another model which considers different limitations of wireless sensor network while generating scale free topology. The probability of new node being damaged (P_e) while deploying is also included in the model. The maximum transmission range of a node is represented by R_{max}. The distance between new node p and existing node q should be less than or equal to R_{max}. There is a degree saturation value for each node represented by D_{max}. The resulting preferential attachment equation is shown in (11)

$$\pi_p = \frac{D_p}{\sum_{q=1}^{N} D_q - nD_{max}} . P\{R_{pq} \leq R_{max}\}. (1 - P_e) \tag{11}$$

The model successfully achieves energy balanced growth of the network. The maximum node degree in a network of 5000 nodes is found to be 180.

Local Area and Energy-Efficient model (LAFE) [43]: In this model, the nodes are dispersed randomly so as to cover the region properly. The nodes get information about their potential neighbours by exchanging hello messages. The model considers node degree and residual energy function of nodes while making preferential attachment as given in (12).

$$\pi_i = \pi_i'(i \in local - area) \frac{f(E_i)k_i}{\sum_{local-area} f(E_j)k_j - qk_{max}} \tag{12}$$

Here, local-area represents the nodes' in the transmission range of new node and K_{max} is the max permissible node degree

Table 2: Comparison of different Scale free models

Researchers/ Model	Topology Information	Mean field analysis	Implementation complexity	Consider Node failures	Application domain
BA Model	global	No	Low	no	www, internet, metabolic network
Link selection model	local	No	Low	no	Internet, Wireless networks
Copying model	local	No	low	no	Citation network, social networks, protein interaction network

Fitness model	global	No	low	no	Social networks, citation network, actor network
Initial attractiveness model	global	No	medium	no	Citation network, actor network, marketing
Zhang Xuyuan[14]	global	Yes	high	yes	Wireless sensor networks
Li-Na Wang et al[15]	Local	No	medium	no	Hierarchical networks
Bing Ye et al [18]	global	Yes	high	no	Economics
Young-bin Kim et al [37]	local	no	medium	no	Wireless networks
XU-Hua Yang et al[40]	Not required	Yes	low	no	Protein interaction networks
Yue Wang et al[42]	local	Yes	medium	yes	Wireless Sensor Networks, P2P networks, Social networks
Hongxing Yao et al [41]	local	Yes	high	yes	Wireless networks, Wireless sensor networks
Yuhui Jian et al [38]	local	Yes	high	no	Wireless sensor networks,
Lurong Jiang et al [43]	local	No	high	no	Wireless networks, Sensor networks
Dong wang et al[45]	Global*	Yes	high	yes	Wireless sensor networks

Table 3: Research areas in scale free networks

Research Areas	Desirable features
Attachment Model	Dynamic, Maintain scale free property, high clustering coefficient, supports node/link addition and deletion

Routing	Small average path length, low communication overhead, minimum network delay, efficient resource utilization
Security	Enhance robustness to withstand and detect malicious attacks, malicious hubs, node failures
Application specific	Context-Aware, features as per resources and limitation of network.
Standard metrics	To identify scale free behaviour, calculate degree exponents
Congestion control	Load balancing, reliability

5. Conclusion and future work

This section discusses some of the open research issues which require further work. Some of the research areas in scale free networks are given in table 3. There are very few standard measures to identify the scale free behaviour in networks and to calculate degree exponents which restrict fair comparison among different scale free models. Many models assume global preference while generating network topology. But, in real world every node cannot connect to any other node due to spatial limitations, bandwidth limitations and energy issues. The preferential attachment in case of social networks is more guided by trust and personality traits rather than by popularity.

There has been a broad range of perspectives on the ubiquitous presence of scale free property in real life networks. There is a group of researchers who identified scale free characteristics in the degree distribution of nodes in various networks as already discussed. On the other hand, another group of researchers contradict the universality of scale free networks. According to Clauset et al. [44], Scale free features are rarely found in real world. They analysed data sets from 927 networks and divided them into several simple graphs by applying few graph transformations. Then, the degree distribution of resulting graphs was compared with several distributions to identify the best fit. On the basis of results obtained, networks were categorized into five classes ranging from super weak to strongest. It was found that only 4% of the networks studied belong to the strongest scale free category. The major drawback of such classification is that the criterion for identifying weak and strong scale free network is somewhat, arbitrary. According to this method, even the pure power-law model fails the strongest scale free test . Moreover, the tests have been applied on finite data sets while scale freeness can be properly explored only in infinite size data sets . A network is usually composed of three degree sequences, in-degree, out-degree and total degree. There may be a case where one of these sequences follows power-law as in the case citation network. According to Matteo Serafino et. al[46], slight deviations from expected power-law behaviour of many networks proposed to be scale free may be due to Finite Size Scaling effects. Scale free networks have emerged as a promising field of network science and more work needs to be done to unlock the potential and opportunities offered by it to solve mysteries of complex world.

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